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THE MONOPHONIC GLOBAL DOMINATION NUMBER OF A GRAPH

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Abstract. A set $M \subseteq V$ is said to be a monophonic global dominating set of G if M is both a monophonic set and a global dominating set of G. The minimum cardinality of a monophonic global dominating set of G is the monophonic global domination number of G and is denoted by $\overline{\gamma}_m(G)$. A monophonic global dominating set of cardinality $\overline{\gamma}_m(G)$ is called a $\overline{\gamma}_m$-*set* of G. The monophonic global domination number of certain classes of graphs are determined. It is proved that $2 \leq \overline{\gamma}_m(G) \leq \overline{\gamma}_g(G) \leq n$, where $\overline{\gamma}_g(G)$ is a geodetic global domination number of a G. It is shown that for every pair of positive integers a and b with $2 \leq a \leq b$, there exists a connected graph G such that $\overline{\gamma}_m(G) = a$ and $\overline{\gamma}_g(G) = b$.

Keywords: monophonic global domination number; global domination number; monophonic number; domination number.

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1. Introduction

By a graph $G = (V, E)$, we mean a finite,undirected connected graph without loops or multiple edges. The *order* and *size* of G are denoted by m and n respectively. For basic graph theoretic terminology, we refer to [2]. Two vertices u and v are said to be *adjacent* if uv is

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an edge of G. If $uv \in E(G)$, we say that u is a *neighbor* of v and denote by $N(v)$, the set of neighbors of v. The *degree* of a vertex $v \in V$ is $deg(v) = |N(v)|$. A vertex v is said to be a *universal vertex* of G if $deg(v) = n-1$. The *subgraph induced* by a set S of vertices of a graph G is denoted by $G[S]$ with $V(G[S]) = S$ and $E(G[S]) = \{uv \in E(G) : u, v \in S\}$. A vertex v is called an *extreme vertex* if $G[N(v)]$ is complete.

The *distance* $d(u,v)$ between two vertices u and v in a connected graph G is the length of a shortest $u-v$ path in G. An $u-v$ path of length $d(u,v)$ is called an $u-v$ *geodesic*. A *distance-hereditary graph* is a graph in which the distances in any connected induced subgraph are the same as they are in the original graph. A vertex x is said to lie on a $u-v$ geodesic P if x is a vertex of P including the vertices u and v. For two vertices u and v, the *closed interval* $I[u,v]$ consists of u and v together with all vertices lying in a u-v geodesic. If u and v are adjacent, then $I[u,v] = \{u,v\}$. For a set S of vertices, let $I[S] = \cup_{u,v \in S} I[u,v]$. Then certainly $S \subseteq I[S]$. A set $S \subseteq V(G)$ is called a *geodetic set* of G if $I[S] = V$. The *geodetic number* $g(G)$ of G is the minimum order of its geodetic sets and any geodetic set of order $g(G)$ is a *geodetic basis* or a *g-set* of G. The geodetic number of a graph was studied in [1,3,5,20-23,26]. A *chord* of a path P is an edge which connects two non-adjacent vertices of P. An u-v path is called a *monophonic path* if it is a chordless path. For two vertices u and v, the *closed interval* $J[u,v]$ consists of all the vertices lying in a u-v monophonic path including the vertices u and v. If u and v are adjacent, then $J[u,v] = \{u,v\}$. For a set M of vertices, let $J[M] = \cup_{u,v \in M} J[u,v]$. Then certainly $M \subseteq J[M]$. A set $M \subseteq V(G)$ is called a *monophonic set* of G if $J[M] = V$. The *monophonic number* $m(G)$ of G is the minimum order of its monophonic sets and any monophonic set of order $m(G)$ is called a *m-set* of G. The monophonic number of a graph was studied in [6,8-19,28].

A subset $D \subseteq V(G)$ is called a *dominating set* if every vertex $v \in V(G) \backslash D$ is adjacent to a vertex $u \in D$. The *domination number*, $\gamma(G)$, of a graph G denotes the minimum cardinality of such dominating sets of G. A minimum dominating set of a graph G is hence often called as a γ-*set* of G. The domination concept was studied in [7]. A subset $D \subseteq V$ is called a *global dominating set* in G if D is a dominating set of both G and $\overline{G}$. The global domination number $\overline{\gamma}(G)$ is the minimum cardinality of a *minimal global dominating set* in G. The concept of global domination in graph was introduced in [25,29,30]. A set $S \subseteq V$ is said to be a geodetic global dominating set of G if S is both a geodetic set and a global dominating set of G. The minimum cardinality of a geodetic global dominating set of G is the geodetic global domination number of G and is denoted by $\overline{\gamma}_g(G)$. A geodetic global dominating set of cardinality $\overline{\gamma}_g(G)$ is called a $\overline{\gamma}_g$-*set* of G. The concept of geodetic global domination in graph was studied in [4,24]. The domination concepts is used in networks. The geodetic and monophonic concepts are used in social networks. By applying the monophonic (geodetic) global domination concepts, there is a effectiveness in the networks. Throughout the following G denotes a connected graph at least two vertices. The following theorem is used in the sequel.

Theorem 1.1. [**4**] Each extreme vertex of a connected graph G belongs to every geodetic global dominating set of G.

2. THE MONOPHONIC GLOBAL DOMINATION NUMBER OF A GRAPH

Definition 2.1. A set $M \subseteq V$ is said to be a monophonic global dominating set of G if M is both a monophonic set and a global dominating set of G. The minimum cardinality of a monophonic global dominating set of G is the monophonic global domination number of G and is denoted by $\overline{\gamma}_m(G)$. A monophonic global dominating set of cardinality $\overline{\gamma}_m(G)$ is called a $\overline{\gamma}_m$-*set* of G.

Example 2.2. For the graph G given in Figure 2.1, $M = \{v_1, v_2, v_5\}$ is a $\overline{\gamma}_m$-set of G so that $\overline{\gamma}_m(G) = 3$.

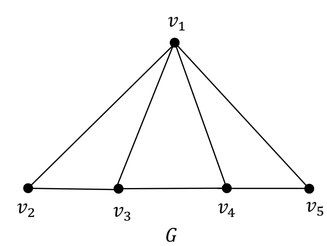

Figure 2.1

Remark 2.3. For the graph G given in Figure 2.1, $M_1 = \{v_2, v_5\}$ is a m-set as well as a γ_m-set of G. Therefore the monophonic global domination number and the monophonic number of a connected graph are different.

Observation 2.4. (i)For a connected graph of order $n \geq 2$, $2 \leq max\{\overline{\gamma}(G), m(G)\} \leq \overline{\gamma}_m(G) \leq n$.
(ii)Each extreme vertex of a connected graph belong to every monophonic global dominating set of G.
(iii)Each universal vertex of a connected graph belong to every monophonic global dominating set of G.

In the following we determine the monophonic global dominating number of some standard graphs.

Theorem 2.5. For the complete graph $G = K_n$ $(n \geq 2)$, $\overline{\gamma}_m(G) = n$.

Proof. This follows from Observation 2.4(ii). □

Theorem 2.6. For the star $G = K_{1,n-1}$ $(n \geq 3)$, $\overline{\gamma}_m(G) = n$.

Proof. This follows from Observation 2.4(ii) and (iii). □

Theorem 2.7. For the the graph $G = K_n - e$ $(n \geq 4)$, $\overline{\gamma}_m(G) = 4$.

Proof. Let $e = uv$. Then u and v are universal vertices of G and $V(G) - \{u, v\}$ is the set of extreme vertices of G. By Observations 2.4(ii) and (iii), $\overline{\gamma}_m(G) \geq n$. Since $V(G)$ is a monophonic global dominating set of G, we have $\overline{\gamma}_m(G) = n$. □

Theorem 2.8. For the path $G = P_n$ $(n \geq 2)$,

$$\overline{\gamma}_m(G) = \begin{cases} 1 & if \quad n \in \{2,3\} \\ \left\lceil \frac{n+2}{3} \right\rceil & if \quad \text{n} \geq 4. \end{cases}$$

Proof. If $n = 2$ or 3, then the result follows from Theorem 2.5 and 2.6. So, let $n \geq 4$. Consider three cases.

Case(a). $n \equiv 0 \pmod 3$. Let $M_1 = \{v_1, v_4, ..., v_{n-2}, v_n\}$. Then M_1 is a minimum monophonic dominating set of G. We show that M_1 is a dominating set of $\overline{G}$. Let $u_i, v_j \in M_1$ be any two adjacent vertices in G. Then $\{v_i, v_j\}$ dominates $\overline{G}$. Since $v_i, v_j \in M_1$, M_1 is a dominating set of $\overline{G}$. By Observation 2.4(ii), M_1 is $\overline{\gamma}_m$-set of G so that $\overline{\gamma}_m(G) = \left\lceil \frac{n+2}{3} \right\rceil$.

Case(b). $n \equiv 1 \pmod 3$. Let $M_2 = \{v_1, v_4, ..., v_{n-3}, v_n\}$. Then by similar argument as in case(a), we can prove that $\overline{\gamma}_m(G) = \left\lceil \frac{n+2}{3} \right\rceil$.

Case(c). $n \equiv 2 \pmod 3$. Let $M_3 = \{v_1, v_4, ..., v_{n-1}, v_n\}$. Then by similar argument as in case(a), we can prove that $\overline{\gamma}_m(G) = \left\lceil \frac{n+2}{3} \right\rceil$. □

Theorem 2.9. For the cycle $G = C_n$ $(n \geq 3)$,

$$\overline{\gamma}_m(G) = \begin{cases} 3 & if \quad n \in \{3,4,5\} \\ \left\lceil \frac{n}{3} \right\rceil & if \quad n \geq 6. \end{cases}$$

Proof. Let C_n be $v_1, v_2, ..., v_n, v_1$. If $n = 3$, then the result follows from Theorem 2.5.

If $n = 4$, then $M_1 = \{v_1, v_2, v_3\}$ is a $\overline{\gamma}_m$-set of G so that $\overline{\gamma}_m(G) = 3$.

If $n = 5$, then $M_2 = \{v_1, v_3, v_4\}$ is a $\overline{\gamma}_m$-set of G so that $\overline{\gamma}_m(G) = 3$.

For $n \geq 6$, we consider three cases.

Case(a). $n \equiv 0 \pmod 3$. Let $M_1 = \{v_1, v_4, ..., v_{n-2}, v_n\}$. Then $J[M_1] = V(G)$ and so M_1 is a monophonic set of G. Since every element of $V(G) - M_1$ is dominated by an element of M_1, M_1 is a dominating set of G. Since G has no chords, any two vertices of M_1 dominates $\overline{G}$ and so M_1 is a global domination set of G. Hence M_1 is a monophonic global set of G and so $\overline{\gamma}_m(G) \leq \left\lceil \frac{n}{3} \right\rceil$. We prove that $\overline{\gamma}_m(G) = \left\lceil \frac{n}{3} \right\rceil$. On the contrary suppose that $\overline{\gamma}_m(G) < \left\lceil \frac{n}{3} \right\rceil$. Then there exists a $\overline{\gamma}_m$-set such that $\mid M' \mid < \left\lceil \frac{n}{3} \right\rceil$. Since $n \geq 6$, there exists at least one $u \in M'$ such that $V(G) - M'$ such that u is not dominated by any element of M'. Therefore M' is not a global domination set of G, which is a contradiction. Therefore M_1 is a $\overline{\gamma}_m$-set of G so that $\overline{\gamma}_m(G) \leq \left\lceil \frac{n}{3} \right\rceil$.

Case(b). $n \equiv 1 \pmod 3$. Let $M_2 = \{v_1, v_4, ..., v_{n-3}, v_n\}$. Then by similar argument as in case(a), M_2 is a $\overline{\gamma}_m$-set of G so that $\overline{\gamma}_m(G) = \left\lceil \frac{n}{3} \right\rceil$.

Case(c). $n \equiv 2 \pmod 3$. Let $M_3 = \{v_1, v_4, ..., v_{n-4}, v_{n-1}\}$. Then by similar argument as in case(a), M_3 is a $\overline{\gamma}_m$-set of G so that $\overline{\gamma}_m(G) = \left\lceil \frac{n}{3} \right\rceil$. □

Theorem 2.10. For the wheel $G = K_1 + C_{n-1}$ $(n \geq 4)$,

$$\overline{\gamma}_m(G) = \begin{cases} 4 & if \quad n \in \{4,5\} \\ 3 & if \quad n \geq 6. \end{cases}$$

Proof. Let $V(K_1) = \{x\}$ and $V(C_{n-1}) = \{v_1, v_2, ..., v_{n-1}\}$. If $n = 4$, then $G = K_4$. Hence the result follows from Theorem 2.5.

If $n = 5$, then $M = \{x, v_1, v_2, v_3\}$ is a $\overline{\gamma}_m$-set of G so that $\overline{\gamma}_m(G) = 4$.

So, let $n \geq 6$. By Observation 2.4(iii), x belongs to every monophonic global dominating set of G. Since $xv_i \in E(G)$ for all i, $(1 \leq i \leq n-1)$, $\overline{\gamma}_m(G) \geq 3$. Let $M_1 = \{x, v_1, v_3\}$. Then M_1 is a $\overline{\gamma}_m$-set of G so that $\overline{\gamma}_m(G) = 3$. □

Theorem 2.11. For the fan graph $G = K_1 + P_{n-1}$ $(n \geq 2)$,

$$\overline{\gamma}_m(G) = \begin{cases} 3 & \text{if } n = 2 \text{ or } n \geq 4 \\ 4 & \text{if } n = 3. \end{cases}$$

Proof. Let $V(K_1) = \{x\}$ and $V(P_{n-1}) = \{v_1, v_2, ..., v_{n-1}\}$. If $n = 2$, then $G = K_3$. Hence the result follows from Theorem 2.7.

If $n = 3$, then $G = K_4 - \{e\}$. Hence the result follows from Theorem 2.5. So, let $n \geq 5$. By Observation 2.4(ii) and (iii), $M = \{x, v_1, v_{n-1}\}$ is a subset of every monophonic global dominating set of G and so $\overline{\gamma}_m(G) \geq 3$. Since M is a monophonic global dominating set of G, we have $\overline{\gamma}_m(G) = 3$. □

Theorem 2.12. For the complete bipartite $G = K_{r,s}$ $(1 \leq r \leq s)$,

$$\overline{\gamma}_m(G) = \begin{cases} s+1 & \text{if} \quad 2 \leq r \leq s \\ r+1 & \text{if} \quad r \leq 3 \\ 4 & \text{if} \quad r \geq 4. \end{cases}$$

Proof. Let $X = \{x_1, x_2, ..., x_r\}$ and $Y = \{y_1, y_2, ..., y_s\}$ be the bipartite sets of G. If $r = 1$, $s \geq 1$, then the result follows from Theorem 2.6. So, let $r \geq 2$. Let M be a monophonic global dominating set of G. Then M contains at least one vertex from X and at least one vertex from Y. Let $2 \leq r \leq 3$. Then $M = X$ is a m-set of G but not a $\overline{\gamma}$-set of G and so $\overline{\gamma}_m(G) \geq r+1$. Let $M_1 = X \cup \{y_1\}$. Then M_1 is a $\overline{\gamma}_m$-set of G so that $\overline{\gamma}_m(G) = r+1$. Let $r \geq 4$. Then $M_2 = \{x_1, x_2, y_1, y_2\}$ is a m-set and a dominating set of G and so $\overline{\gamma}_m(G) \geq 4$. Since M_2 is a $\overline{\gamma}$-set of G, M_2 is also a $\overline{\gamma}_m$-set of G so that $\overline{\gamma}_m(G) = 4$. □

Theorem 2.13. For the helm graph $G = H_r$, $\overline{\gamma}_m(G) = r+1$.

Proof. Let x be the central vertex of G and Z be the set of r end vertices of G. By Observation 2.4(ii), Z is a subset of every monophonic global dominating set of G. Since x is not dominated by any vertex of Z, Z is not a monophonic global dominating set of G and so $\overline{\gamma}_m(G) \geq r+1$.

Let $Z' = Z \cup \{x\}$. Then $J[Z'] = V(G)$ and every element of $V(G) - Z'$ is dominated by at least one element of Z'. Therefore Z' is a monophonic global dominating set of G so that $\overline{\gamma}_m(G) = r+1$. □

Theorem 2.14. For the banana tree graph $G = B_{r,s}$, $\overline{\gamma}_m(G) = r+1$.

Proof. Let x be the central vertex of G and Z be the set of end vertices of G. By Observation 2.4(ii), Z is a subset of every monophonic global dominating set of G. Since x is not dominated by any vertex of Z, Z is not a monophonic global dominating set of G and so $\gamma_m(G) \geq r+1$. Let $Z' = Z \cup \{x\}$. Then $J[Z'] = V(G)$ and every element of $V(G) - Z'$ is dominated by at least one element of Z'. Therefore Z' is a monophonic global dominating set of G so that $\overline{\gamma}_m(G) = r+1$. □

3. The Monophonic Global Domination Number and the Geodetic Global Domination of a Graph

Theorem 3.1. Every geodetic global dominating set of G is a monophonic global dominating set of G.

Proof. Let S be a geodetic global dominating set of G. Then S is a geodetic set and a global dominating set of G. Since every u-v geodesic is a u-v monophonic path, S is a monophonic set of G. Hence it follows that S is a global dominating set of G. □

Let G be a connected graph of order n. Then $2 \leq \overline{\gamma}_m(G) \leq \overline{\gamma}_g(G) \leq n$.

Proof. This follows from Theorem 2.8. □

Theorem 3.2. Let G be a distance-hereditary graph of order n. Then $\overline{\gamma}_g(G) = \overline{\gamma}_m(G)$.

Proof. In a distance-hereditary graph, every u-v monophonic path is a u-v geodesic. Hence it follows that every monophonic global dominating set of G is a geodetic global dominating set of G. Therefore $\overline{\gamma}_g(G) \leq \overline{\gamma}_m(G)$. Hence the result follows from Corollary 3.2. □

In the view of Corollary 3.2, we have the following realization result.

Theorem 3.3. For every pair of positive integers a and b with $2 \leq a \leq b$, there exists a connected graph G such that $\overline{\gamma}_m(G) = a$ and $\overline{\gamma}_g(G) = b$.

Proof. For $a = b$, let $G = K_{1,a-1}$. Then the result follows from Theorem 2.6. So, let $2 \leq a < b$. Let $P : x, y, z$ be a path on three vertices. Let $P_i : u_i, v_i$ $(1 \leq i \leq b-a+1)$ be a copy of path on two vertices. Let H be a graph obtained from P and P_i $(1 \leq i \leq b-a+1)$ by introducing the edges xu_i and zv_i $(1 \leq i \leq b-a+1)$. Let G be the graph obtained from H by adding new vertices $z_1, z_2, ..., z_{a-2}$ and introducing the edges zz_i $(1 \leq i \leq a-2)$. The graph G is shown in Figure 3.1.

First we prove that $\overline{\gamma}_m(G) = a$. Let $Z = \{z_1, z_2, ..., z_{a-2}\}$ be the set of all end vertices of G. By Observation 2.4(ii), Z is a subset of every monophonic global dominating set of G and so $\overline{\gamma}_m(G) \geq a-2$. Since $J[Z] \neq V$, Z is not a monophonic global dominating set of G and so $\overline{\gamma}_m(G) \geq a-1$. It is also noted that $Z \cup \{u\}$, where $u \notin Z$ is not a global dominating set of G and so $\overline{\gamma}_m(G) \geq a$. Let $M = Z \cup \{x, z\}$. Then M is a $M = Z \cup \{x, z\}$. Then M is a monophonic global dominating set of G so that $\overline{\gamma}_m(G) = a$.

Next we prove that $\overline{\gamma}_g(G) = b$. By Theorem 1.1, Z is a subset of every geodetic global dominating set of G. Let $H_i = \{u_i, v_i\}$ $(1 \leq i \leq b-a+1)$. It is noted that every minimum global dominating set of G contains exactly one vertex from each H_i $(1 \leq i \leq b-a+1)$ and either x or y and so $\overline{\gamma}_g(G) \geq a-2+b-a+1+1 = b$. Let $S = Z \cup \{x, v_1, v_2, ..., v_{b-a+1}\}$. Then S is a geodetic global dominating set of G so that $\overline{\gamma}_g(G) = b$. □

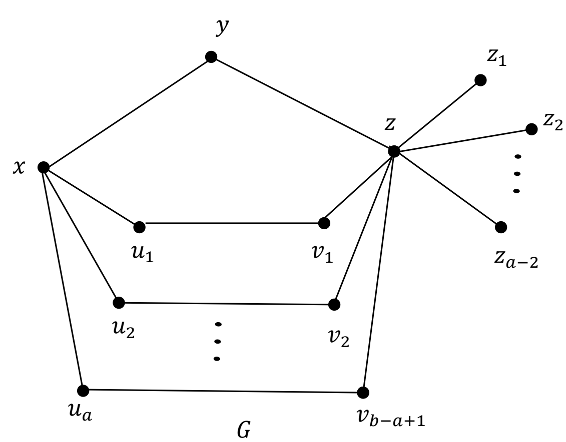

Figure 3.1

4. Conclusion

In this article, we introduced the concept of the monophonic global domination number of a graph and studied some of its general properties. It can be further investigated to find out under which conditions the lower bound and the upper bound of the monophonic global domination number are sharp.

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Conflict of Interests

The author(s) declare that there is no conflict of interests.

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